Solutions to Workbook-2 [Mathematics] | Permutation & Combination

JEE Archive	DAILY TUTORIAL SHEET 2

13.(35) Since, six '+' signs are + + + + + + $\therefore$ 4 negative sign has seven places to be arranged in

$\Rightarrow$ ${}^{7}C_{4}$ ways = 35 ways

14.(9) The number of ways in which the ball does not go its own colour box

$= 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) = 4!\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right) = 24\left(\frac{12-4+1}{24}\right) = 9$

15.(C) 0 identical + 10 distincts, number of ways = $1\times {}^{21}C_{10}$

1 identical + 9 distincts, number of ways = $1\times {}^{21}C_{9}$

2 identical + 8 distincts, number of ways = $1\times {}^{21}C_{8}$

So, total number of ways in which we can choose 10 objects is

${}^{21}C_{10} + {}^{21}C_{9} + {}^{21}C_{8} + ... + {}^{21}C_{0} = x(\text{let})$ ...(i)

$\Rightarrow$ ${}^{21}C_{11} + {}^{21}C_{12} + {}^{21}C_{13} + ... + {}^{21}C_{21} = x$ ...(ii)

$\left[\because\ {}^{n}C_{r} = {}^{n}C_{n-r}\right]$

On adding both Eqs. (i) and (ii), we get

$2x = {}^{21}C_{0} + {}^{21}C_{1} + {}^{21}C_{2} + ... + {}^{21}C_{10} + {}^{21}C_{11} + {}^{21}C_{11} + {}^{21}C_{12} + ... + {}^{21}C_{21}$ $\Rightarrow$ $2x = 2^{21}$ $\Rightarrow$ $x = 2^{20}$

16.(C) The top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then total number of beams = number of diagonals of 20-sided polygon.

$\therefore$ ${}^{20}C_{2}$ is selection of any two vertices of 20-sided polygon which included the sides as well.

So, required number of total beams = ${}^{20}C_{2} - 20$

[$\because$ the number of diagonals in a n-sided closed polygon = ${}^{n}C_{2} - n$] = $\frac{20\times 19}{2} - 20 = 190 - 20 = 170$

17.(B) According to the question, we have $\frac{n(n+1)}{2} + 99 = (n-2)^{2}$ $\Rightarrow n = 19$ [$\because$ number of balls n > 0]

Now, number of balls used to form an equilateral triangle is $\frac{n(n+1)}{2} = \frac{19\times 20}{2} = 190.$

18.(A) Number of games played by the men between themselves $= 2\times {}^{m}C_{2}$

And the number of games played between the men and the women = $2\times {}^{m}C_{1}\times {}^{2}C_{1}$

Now, according to the question, $2\ {}^{m}C_{2} = 2\ {}^{m}C_{1}\ {}^{2}C_{1} + 84 \Rightarrow \frac{m!}{2!(m-2)!} = m\times 2 + 42$ $\Rightarrow$ $m = 12$

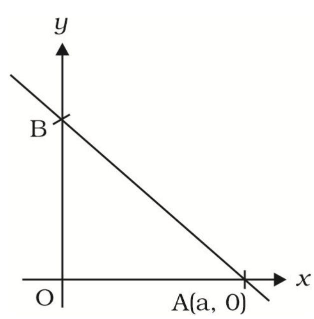
19.(C) The number of ways, in which a team of 3 students can be selected from this group

= (number of ways selecting one boy and 2 girls) + (number of ways selecting two boys and 1 girl)

$= \left({}^{5}C_{1}\times {}^{n}C_{2}\right) + \left({}^{5}C_{2}\times {}^{n}C_{1}\right) = 1750$ [given]

$\Rightarrow$ $\left(5\times\frac{n(n-1)}{2}\right) + \left(\frac{5\times 4}{2}\times n\right) = 1750$ $\Rightarrow$ $\boxed{n = 25}$

20.(A) Here O(0,0), $A(\alpha,0)$ *and* $B(0,b)$ are the three vertices of the triangle.



Clearly, $OA = |a|$ *and* $OB = |b|$. $\therefore$ Area of $\Delta OAB = \frac{1}{2}|a||b|$.

But area of such triangle is given as 50 sq units.

$\therefore$ $\frac{1}{2}|a||b| = 50$ $\Rightarrow$ $|a||b| = 100 = 2^{2}.5^{2}$

Number of ways of distributing two $2's$ in $|a|$ and $|b| = 3$

$\|a\|$	$\|b\|$
0	2
1	1
2	0

$\Rightarrow$ 3 ways

Similarly, number of ways of distributing two $5's\ in\ |a|\ and\ |b| = 3$ ways.

$\therefore$ Total number of ways of distributing $2's\ and\ 5's = 3 \times 3 = 9$ ways

For one value of $|a|$, there are 2 possible values of a and for one value of $|b|$, three are 2 possible values of b. $\therefore$ Number of such triangles possible = 2 × 2 × 9 = 36.

So, number of elements in S is 36.

21.(A) Since, the sum of given digits 0+1+2+5+7+9 = 24

Let the six-digit number be *abcdef* and to be divisible by 11, $(a+c+e)-(b+d+f)$ should be either 0 or a multiple of 11. Hence, possible case is $a+c+e = 12 = b+d+f\ (only)$

Now, Case I

Set $\{a, c, e\} = \{0, 5, 7\}\ and\ set\ \{b, d, f\} = \{1, 2, 9\}$

So, number of 6-digits numbers = $(2 \times 2!) \times (3!) = 24$

[$\because$ a can be selected in ways only either 5 or 7].

Case II

Set $\{a, c, e\} = \{1, 2, 9\}\ and\ set\ \{b, d, f\} = \{0, 5, 7\}$

So, number of 6-digits numbers = $3! \times 3! = 36$ So, total number of 6-digits numbers = 24 + 36 = 60

22.(C) $m =$ number of ways when there is at least 6 males $= \left({}^8C_6 \times {}^5C_5\right) + \left({}^8C_7 + {}^5C_4\right) + \left({}^8C_8 \times {}^5C_3\right) = 78$

and n = number of ways when there is at least 3 females

$= \left({}^5C_3 \times {}^8C_8\right) + \left({}^5C_4 + {}^8C_7\right) + \left({}^5C_5 \times {}^8C_6\right) = 78$ So, $m = n = 78$

23.(B) One ball is randomly drawn from each boxes, and n_i denote the label of the ball drawn from the ith box, $(i-1, 2, 3)$.

Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is same as selection of 3 different numbers from numbers $\{1, 2, 3, \ldots\ldots, 10\} = {}^{10}C_3 = 120$.

24.(A) Using the digits 0, 1, 3, 7, 9

Number of one-digit natural numbers that can be formed = 4,

Number of two-digit natural numbers that can be formed = 20

4 × 5

($\because$ 0 can-not come in Ist box)

Number of three-digit natural numbers that can be formed = 100

4 × 5 × 5

and number of four-digit natural numbers less than 7000, that can be formed = 250

2 × 5 × 5 × 5

($\because$ only 1 or 3 can come in Ist box)

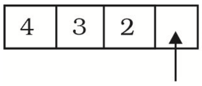
$\therefore$ Total number of natural numbers formed = 4 + 20 + 100 + 250 = 374

25.(D) Now, total ways of forming a team of 3 boys and 2 girls = $= {}^7C_3 . {}^5C_2 = 350$

But, if two specific boys are in team, then number of ways = ${}^5C_1 . {}^5C_2 = 50$

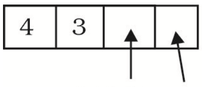
Required ways, i.e., the ways in which two specific boys are not in the same team = 350 – 50 = 300

26.(B) **Case-I**



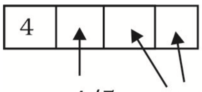
2/3/4/5 → 4 ways   4 numbers

Case-II



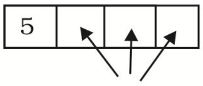
3/4/5 3 ways   0/1/2/3/4/5 6 ways $3 \times 6 = 18$ numbers

Case-III



4/5 2 ways   0/1/2/3/4/5 6 ways $2 \times 6 \times 6 = 72$ numbers

Case-IV



0/1/2/3/4/5 6 ways $6 \times 6 \times 6 = 216$ numbers

So, required total numbers = 4 + 18 + 72 + 216 = 310